**Analytical Solution for Heat Transfer in Electroosmotic Flow of a Powell Eyring Fluid in a Wavy Microchannel**

Project report submitted

in partial fulfilment of the requirement for the degree of

**Bachelor of Technology**

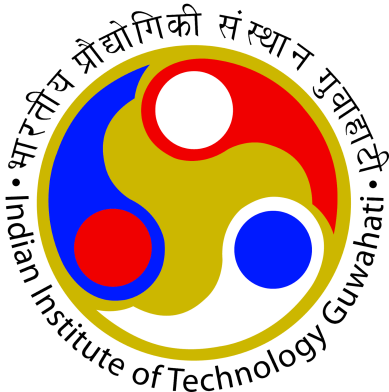
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**Abstract**

We investigate the heat and transport characteristics of electroosmotic flow augmented with peristaltic transport of incompressible Powell-Eyring fluid through a wavy microchannel. In order to determine the energy distribution, viscous dissipation is reckoned. Debye Hückel linearization and long wavelength assumptions are adopted. Resulting non-linear problem is solved by homotopy perturbation method (HPM), to examine the distribution and variation in velocity, temperature, and volumetric flow rate within the Powell-Eyring fluid flow pattern through perturbation technique. This model is also suitable for a wide range of biological microfluidic applications and variation in velocity, temperature, and volumetric flow rate within the Powell-Eyring fluid flow pattern.

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**1. Introduction**

The peristaltic transport in fluid mechanics has a lot of significance in biological science and hydrodynamics. A peristaltic transport is created by wave propagation along the flexible wall of the channel. This mechanism exists in many biological systems with smooth muscles tubes e.g., the movement of food in the digestive tract, urine transport, blood circulation in small blood vessels, etc. Electroosmosis is one of the main electrokinetic phenomena. Electroosmosis refers to the flow of liquid in contact with a charged substrate upon the application of an external electric field due to resultant force acting on the ions present in the liquid. The ionic motion drags liquids along and there is fluid flow by virtue of fluid viscosity called as Electroosmotic transport. Such flows, with uniform interfacial potential and with thin electrical double layer (EDL), are typically characterized by a plug profile like flow having a characteristic velocity termed as the Helmholtz-Smoluchowski velocity, a characteristic feature of Newtonian fluids. Electroosmosis of non-Newtonian fluids has gained a significant research attention over recent years. Miniaturization, integration, lack of moving parts and commitment of minimal sample dispersion have promoted the use of electroosmotic effects in actuating flows of non-Newtonian fluids in several Lab-on-a-Chip based bio-microfluidic applications such as drug delivery, chemical analysis, mixing etc., particularly, for the analysis of bio-fluids such as blood and DNA solutions.

There are different models of non-Newtonian fluids like power law model, carreau model, etc but to study the rheological behaviour of the non-Newtonian fluids a single constitutive relationship between shear stress and shear rate cannot be used as they become Newtonian for a particular value of index. Powell-Eyring model overcomes this limitation, it accurately represents the constitutive behaviour in describing the flow behaviour of non-Newtonian fluids at both the low and high shear rates. Although this model has complexity in mathematical analysis, it purely a non-Newtonian model as it is deduced from the kinetic theory of fluids instead of using any empirical relation like power-law model which reduces to the Newtonian behaviour at low and high shear rates. Here, we study different attributes governing electroosmosis of Powell-Eyring fluid in a slit microchannel, and bring out the alteration in flow dynamics as modulated by the interactions between flow rheology, electrostatics within the EDL, and interfacial slip. Although the molecular packing in liquids is dense enough to rule out interfacial slip in many practical scenarios, slip in liquids cannot be trivially precluded, especially considering transport phenomena over small scales. True interfacial slip may occur in scenarios in which molecular interactions cause a relative motion between the fluid molecules and the solid boundary at their points of contact. Such phenomena may be possible at very high shear rates, so that liquid molecules may be sheared away from the bounding solid substrates overcoming the attractive interactions. On the other hand, an apparent interfacial slip may occur when microfluidic or nanofluidic substrates are covered with a low-density depleted fluid layer (such as nanobubble layer), as attributable to complex hydrophobic interactions. Such phenomena may be typical to microfluidic or nanofluidic substrates of lab on a chip based miniaturized devices handling complex bio-fluids.

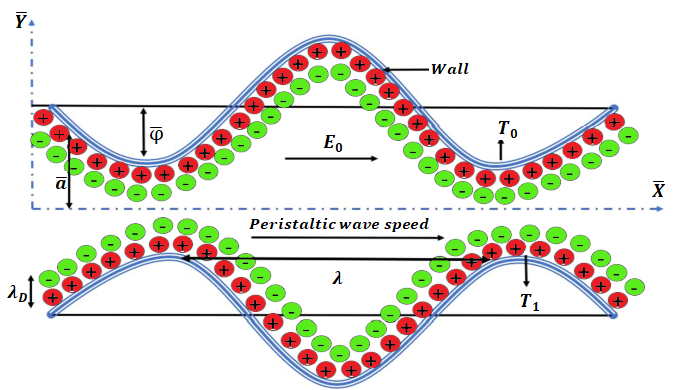
A regular perturbation method is used to solve the current problem. A complete parametric study is performed to find out the effects of parameter R, electroosmotic parameter me, Helmholtz–Smoluchowski velocity UHS, volume flow rate θ and Brinkman number Br, axial velocity u, pressure gradient on the thermal features of the flow.

**2. Literature Review**

Noreen at el. [1] analyzed the heat transfer of carreau fluid in a wavy micro-channel. Prakash Goswami, Pranab Kumar Mondal, Sanmitra Dutta and Suman Chakraborty [2] have studied Electroosmosis of powell-eyring fluids under interfacial slip. Noreen and Saleem [3] analyzed the impact of the magneto-hydrodynamic peristaltic transport for Dufour and Soret effects by using thermal radiation and chemical reaction in a porous medium. Noreen [4] examined the electro-magneto-hydrodynamic peristaltic transport of Couple-Stress fluid along Joule heating and convective boundary conditions. Moreover, Makinde et al. [5] proposed the peristaltic motion model for walters-b fluid with thermal radiations and slip conditions. The peristaltic modulation of Jeffrey fluid in an inclined channel with a Newtonian fluid was scrutinized by Kavitha et al. [6]. The peristaltic flow of Eyring-Powell nanofluid model in the asymmetric conduit was investigated by Noreen [7]. Also, Ijaz et al. [8] explained the propulsion of Ree-Eyring fluid through a peristaltic duct. Mekheimer et al. [9] found the gold nanoparticles in the catheter as a third grade nanofluid. Manjunatha et al. [10] considered that the slip effects in an inclined peristaltic tube for Casson fluid. Chakraborty [12] analyzed the capillary transport of biofluid in a microchannel through electroosmosis. Gao et al. [13] discussed the EOF of two-liquid flow via microchannel. Zhao et al. [14] demonstrated the EOF of power-law fluid through slit microchannel. Also, Tang et al. [15] studied non-Newtonian fluids in the microchannel. Moreover, Vasu et al. [16] explored the electroosmotic influence of Power-law fluid for different zeta potential. Hadigol et al. [17] explained the micro-mixing (electroosmotic) of non-Newtonian fluids, numerically. Also, Choi et al. [18] inspected the EOF of viscoelastic fluid in an asymmetric channel. Zhao and Yang [19] observed the EOF of power-law fluid through the cylindrical microchannel. Yavari et al. [20] demonstrated temperature rise for biofluids in EOF. Qi and Ng [21] assumed the effect of wall potential and variation in height of channel for EOF of power-law fluid through slit microchannel. Kung et al. [22] examined 3D microfluidic network with PDMC surface and hybrid stamp. Tripathi et al. [23] observed the EDL effects in peristaltic transport. Kung et al. [24] studied the mechanism of Tunnel Di-electrophoresis in high-speed microfluidic flows. Also, Bhatti et al. [25] briefed the analysis of entropy for magnetized nanofluid peristaltic flow via microchannel. EOF of Williamson ionic nanofluid peristaltic transport through a microchannel was incorporated by Prakash and Tripathi [26]. Tripathi et al. [27] also presented the impacts of blood flow as a non-Newtonian fluid in EOF.

**3. Problem Formulation**

Consider a two-dimensional peristaltic flow of an incompressible Powell Eyring fluid through a way micro-channel as shown in fig. 1.



**Fig. 1. Problem Configuration**

The wall deformation in the micro-channel is given by, [23]

(1)

where ,, , , , and are the axial coordinate, transverse vibration of the wall, wave amplitude, channel’s half width, wave speed, time and wavelength respectively.

**3.1 Governing Equations**

The governing equations for mass, momentum and energy in laboratory frame are [1]:

Continuity equation:

(2)

Momentum equation in x-direction:

(3)

Momentum equation in y-direction:

(4)

Energy equation:

(5)

Stress tensor for Powell Eyring fluid is given by [2],

(6)

Where, are velocity components along direction respectively. Similarly, ,, *,* and B & C represents the density of the fluid, electrical charge density, axial electric field, pressure, temperature, thermal conductivity of the fluid, Cauchy stress tensor, coefficient of viscosity and material fluid parameters respectively.

To shift from laboratory frame to wave frame , we define the translational transformation as [8]:

For the simplification of governing equations, consider:

(7)

Here, x & y are the non-dimensional axial coordinate & transverse coordinate, h is the non-dimensional transverse wall’s vibration,is the non-dimensional wave amplitude,is non-dimensional pressure, is electroosmotic parameter, α is peristaltic wave number, is characteristic thickness of EDL/Debye length, is Helmholtz–Smoluchowski velocity, is non-dimensional stream function, is Peclet number, is Reynolds number, is Brinkman number, is Prandtl number, is Eckert number, is dimensionless volume flow rate.

Assuming second term of stress tensor can be expanded as, [28-30, 42]

(8)

Hence, using the assumption considered in Eq. (8), i.e., the stress tensor can be written as:

(9)

**3.2 Non – Dimensionless form of governing equations** [2]

Using equations (7) and (9), the non-dimensional form of governing equations (3)-(5) are,

(10)

(11)

(12)

After cross differentiation and eliminating pressure terms from (10) and (11) we get,

(13)

The electric potential distribution within a microchannel is given by Poisson-Boltzmann equation [12]:

(14)

Here represents permittivity of free space, total charge density and relative permittivity of the medium. For electrolyte symmetry assumption, total charge density is taken:

(15)

where the cations can be described by number density of Poisson equation:

(16)

The non-dimensional equations of (14) & (16) are,

(17)

(18)

Substituting (17) in (16) gives,

(19)

Using Debye-Hückel linearization approximation [16] i.e., sinh ≈ , the above equation becomes,

(20)

Whereas the analytical solution of electrical potential is given by,

**4. Solution Methodology**

Equation (13) is a nonlinear PDE, and its closed form solution is not possible. Therefore, it can be solved analytically by perturbation technique about parameter, by expanding in the following forms [25]:

(19)

(20)

(21)

(22)

**5. Results**

**6. Conclusions**

**7. Future Work**

**8. References**

1. Saima Noreen, Sadia Waheed, Abid Hussanan and Dianchen, Analytical Solution for Heat Transfer in Electroosmotic Flow of a Carreau Fluid in a Wavy Microchannel.
2. Prakash Goswami, Pranab Kumar Mondal, Sanmitra Dutta and Suman Chakraborty, Electroosmosis of powell-eyring fluids under interfacial slip.
3. Noreen, S.; Saleem, M. Soret and Dufour effects on the MHD peristaltic flow in a porous medium with thermal radiation and chemical reaction. Heat Transf. Res. 2016, 47.
4. Noreen, S. Effects of Joule Heating and Convective Boundary Conditions on Magnetohydrodynamic Peristaltic Flow of Couple-Stress Fluid. J. Heat Transf. 2016, 138, 094502.
5. Makinde, O.D.; Reddy, M.G.; Reddy, K.V. Effects of thermal radiation on MHD peristaltic motion of walters-b fluid with heat source and slip conditions. Energy 2017, 5, 7.
6. Kavitha, A.; Reddy, R.H.; Saravana, R.; Sreenadh, S. Peristaltic transport of a Jeffrey fluid in contact with a Newtonian fluid in an inclined channel. Ain Shams Eng. J. 2017, 8, 683–687.
7. Noreen, S. Magneto-thermo hydrodynamic peristaltic flow of Eyring-Powell nanofluid in asymmetric channel. Nonlinear Eng. 2018, 7, 83–90.
8. Ijaz, N.; Zeeshan, A.; Bhatti, M.M. Peristaltic propulsion of particulate non-Newtonian Ree-Eyring fluid in a duct through constant magnetic field. Alex. Eng. J. 2018, 57, 1055–1060.
9. Mekheimer, K.S.; Hasona, W.M.; Abo-Elkhair, R.E.; Zaher, A.Z. Peristaltic blood flow with gold nanoparticles as a third grade nanofluid in catheter: Application of cancer therapy. Phys. Lett. A 2018, 382, 85–93.
10. Manjunatha, G.; Choudhary, R.V. Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with porous walls. J. Adv. Res. Fluid Mech. Therm. Sci. 2018, 43, 67–80.
11. Chakraborty, S. Augmentation of peristaltic microflows through electro-osmotic mechanisms. J. Phys. D Appl. Phys. 2006, 39, 5356.
12. Chakraborty, S. Electroosmotically driven capillary transport of typical non-Newtonian biofluids in rectangular microchannels. Anal. Chim. Acta 2007, 605, 175–184.
13. Chakraborty, S. Electroosmotically driven capillary transport of typical non-Newtonian biofluids in rectangular microchannels. Anal. Chim. Acta 2007, 605, 175–184.
14. Zhao, C.; Zholkovskij, E.; Masliyah, J.H.; Yang, C. Analysis of electroosmotic flow of power-law fluids in a slit microchannel. J. Colloid Interface Sci. 2008, 326, 503–510.
15. Tang, G.H.; Li, X.F.; He, Y.L.; Tao, W.Q. Electroosmotic flow of non-Newtonian fluid in microchannels. J. Non-Newton. Fluid Mech. 2009, 157, 133–137.
16. Vasu, N.; De, S. Electroosmotic flow of power-law fluids at high zeta potentials. Colloids Surf. A Physicochem. Eng. Asp. 2010, 368, 44–52.
17. Hadigol, M.; Nosrati, R.; Nourbakhsh, A.; Raisee, M. Numerical study of electroosmotic micromixing of non-Newtonian fluids. J. Non-Newton. Fluid Mech. 2011, 166, 965–971.
18. Choi, W.; Joo, S.W.; Lim, G. Electroosmotic flows of viscoelastic fluids with asymmetric electrochemical boundary conditions. J. Non-Newton. Fluid Mech. 2012, 187, 1–7.
19. Zhao, C.; Yang, C. Electroosmotic flows of non-Newtonian power-law fluids in a cylindrical microchannel. Electrophoresis 2013, 34, 662–667.
20. Yavari, H.; Sadeghi, A.; Saidi, M.H.; Chakraborty, S. Temperature rise in electroosmotic flow of typical non-newtonian biofluids through rectangular microchannels. J. Heat Transf. 2014, 136, 031702.
21. Qi, C.; Ng, C.O. Electroosmotic flow of a power-law fluid in a slit microchannel with gradually varying channel height and wall potential. Eur. J. Mech. B Fluids 2015, 52, 160–168.
22. Kung, Y.C.; Huang, K.W.; Fan, Y.J.; Chiou, P.Y. Fabrication of 3D high aspect ratio PDMS microfluidic networks with a hybrid stamp. Lab Chip 2015, 15, 1861–1868.
23. Tripathi, D.; Bhushan, S.; Bég, O.A. Transverse magnetic field driven modification in unsteady peristaltic transport with electrical double layer effects. Colloids Surf. A Physicochem. Eng. Asp. 2016, 506, 32–39.
24. Kung, Y.C.; Huang, K.W.; Chong, W.; Chiou, P.Y. Tunnel Dielectrophoresis for Tunable, Single-Stream Cell Focusing in Physiological Buffers in High-Speed Microfluidic Flows. Small 2016, 12, 4343–4348.
25. Bhatti, M.M.; Sheikholeslami, M.; Zeeshan, A. Entropy analysis on electro-kinetically modulated peristaltic propulsion of magnetized nanofluid flow through a microchannel. Entropy 2017, 19, 481.
26. Prakash, J.; Tripathi, D. Electroosmotic flow of Williamson ionic nanoliquids in a tapered microfluidic channel in presence of thermal radiation and peristalsis. J. Mol. Liq. 2018, 256, 352–371.
27. Tripathi, D.; Yadav, A.; Bég, O.A.; Kumar, R. Study of microvascular non-Newtonian blood flow modulated by electroosmosis. Microvasc. Res. 2018, 117, 28–36.